Theoretical Hydrodynamic Coefficients of Laterally Oscillating Profiles at High Froude Number

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Surface effect ships are envisaged which may exceed 100 knots speeds and 10,000 tons weight. Their lightweight structure leads to the possibility of hydroelastic instabilities and dynamic amplification of loads due to waves. Literature surveys indicate that hydrodynamic coefficients for lateral oscillations of slender profiles at high Froude number do not appear to have been studied. This paper describes initial phases of the determination of theoretical coefficients as an expansion in inverse Froude number, for moderate draft to length ratio, and very low frequency lateral motions. Integral equations are formulated for various orders of terms in a series expansion in inverse Froude number for hydrodynamic coefficients due to sway, roll, and yaw of a profile. Zeroth-order terms are determined explicity, first-order terms implicitly. Explicit first-order terms will be the subject of a subsequent paper.

Nomenclature

A_{ij}, D_{ij}, K_{ij} = hydrodynamic inertia, damping and stiffness coefficients
$C_n, {}^{()}C, {}^{()}C_n$ = numerical constants
$d(x_l)$ = profile local depth
$E_n(z)$ = exponential integrals
$f(x), g(x)$ = arbitrary functions
F = Froude number
g = gravitational constant
$G(x_i; y_i)$ = Green's function
L_J = wake integral Eq. (40)
I, I_β = imaginary part of M, M_β
k, k_1, k_2 = dimensionless wave numbers
K = $F^{-1/2}$, expansion variable
ℓ = profile length
L = integration path
M, M_β = doublet distribution, ditto in mode β
$\eta_{\beta+}$ = unit normal
${}^{()}N_\beta$ = "boundary conditions" of order ( )
p = $\bar{p} \div \rho u^2/2$ dimensionless perturbation pressure
$\bar{p}$ = perturbation pressure
Q_i = generalized force Eq. (50)
R, R_β = real part of M, M_β
S_i = various surfaces
t = time
U = undisturbed fluid velocity
V_i = fluid velocity vector (perturbation)
W = work
χ = modulus of z
X_i, x_i = dimensional and dimensionless coordinates
$\bar{x}_3, \bar{y}_3$ = $x_{3/\epsilon}, y_{3/\epsilon}$
z = $k^2 \sec^2\Theta \{(x_3 + y_3) + i(x, -y,)\cos\Theta\} = \chi \exp(i\textcircled{\Theta})$
$\gamma, \tilde{\gamma}$ = $-(x_3 + y_3)/\epsilon, -(x_3 + \xi_3)/\epsilon$ [Eqs. (20), (42)
$\bar{\gamma}$ = Euler's constant
$\Gamma(z)$ = gamma function Eq. (24)
$\delta(\ )$ = delta function
ϵ = maximum depth
ζ_β = perturbation freedoms
η = $(\sec^2\Theta - 1)$
Θ = phase variable
$\textcircled{\Theta}$ = argument of z
ν = reduced frequency
ξ_3 = a dummy y_3 variable
ρ = density of water
τ = dimensionless time $= tU/\ell$
ϕ = dimensionless velocity potential, $\Phi \div U\ell$
Φ = velocity potential
$\psi(\ )$ = logarithmic derivative of the gamma function
ω = oscillation angular frequency
$()_{,i}\ ()_{,ii}$ = typically $\partial/\partial x_i$, $\sum_i \partial^2/\partial x_i^2$
$(\dot{\ })$ = time derivatives, $d/d\tau$
S_θ' = one side of a surface S_i
ℓ, t = subscripts for leading and trailing edges
$+, -$ = subscription for $+x_i$ and $-x_i$ sides of S, p, etc.
O, R, L, W = superscripts for asymptotic, residual, local and wave functions
β = subscript for mode number

Introduction

THE structures of surface effect ships (SES) are quite unlike buoyed ships, in that lightness is at a premium. Efficient structural practices and aluminum alloys are essential design features as in aircraft design.[1] The SES theoretically rides clear of the water, but craft motion and waves create intermittent partial structural immersion. Dynamic stability, response, and load problems are complicated by such departures from ideal operating conditions.

The fluid structure interaction terms are proportional to the dynamic pressure of the relative flow, $\frac{1}{2}\rho U^2$, in which at 100 knots, the SES experiences more than an order of magnitude increase (typically to 25000 psf) compared with buoyancy vessels. This, taken with their relatively lightweight structure, introduces the possibility of new problems of hydroelastic

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stability due to interaction between motions of the small submerged depth of sidewall and the relative flow. Such effects may play a significant role in structural stiffness requirements (and hence weight and economics) of projected larger SES.

All structures possess dynamic inertia and flexibility properties related to material constitutive properties and general structural design features. There is a long history of problems due to static and dynamic interactions between structures and a relative fluid flow.[2,3] The classical problem is that of aircraft flutter, involving feedback between unsteady fluid dynamic forces and the structural oscillations. These problems become more important as size and speed increase, when stiffness and mass distribution play an increasingly critical role in the occurrence and cure of fluid-elastic instabilities. On a stable structure, the margin of stability strongly influences the magnitude of displacement and force response, resulting from external disturbances (e.g., gusts on aircraft, waves on ships).

SES sidewalls are typically long and slender, a feature (exploited in slender-body aerodynamic theory, ship hydrodynamic theory, and in the present work) permitting linearization and the separation of "thickness' and "camber" problems.[4] Newman[5] has given a thorough survey of applications of slender body theory in ship hydrodynamics, but no treatment of lateral coefficients for high Froude numbers appears to exist.

The purpose of this study was to develop hydrodynamic coefficients for lateral motions of such sidewalls at high Froude number for subsequent stability studies. An idealized problem is solved as a first step. We study the hydrodynamic effects of low frequency lateral oscillations of a thin vertical structure of arbitrary shape, moving forward at a high constant speed, and piercing the free surface of a semi-infinite fluid. (Fig. 1).

The problem is linearized and formulated as an integral equation on a nondimensional velocity potential as a doublet distribution on the profile. An appropriate Green's function is found and is expanded in powers of inverse Froude number, as is the velocity potential. This is then used to formulate sequential integral equations and boundary conditions for various orders of solution. Zero-order solutions are presented explicitly, first-order solutions are presented implicitly. Subsequent papers will treat explicit first-order coefficients, the incorporation of sidewall thickness, and applications in structural hydroelastic stability.

This investigation neglects cavitation and viscous effects. The need for corrections for these effects should be explored experimentally at a later stage.

Fundamental Equations

Differential Equations and Boundary Conditions

A rigid plate of maximum length ℓ in the X_1 direction and maximum draft $\epsilon\ell$ in the X_3 direction moves forward ($+X_1$ direction) at a constant speed U through a fluid with undisturbed free surface $X_3=0$. The plate is assumed to be so thin that it may be replaced by its profile, $x_3=-d(x_1)$, of zero thickness, i.e. the "camber" problem is separated from the "thickness" problem[4,7], by linearization of the problem. The coordinate system is attached to the plate. The profile undergoes small lateral time harmonic oscillations defined as a sidewards translation (sway), $\zeta_2\ell \exp(i\omega t)$ along the X_2 axis, and rotations, roll, $\zeta_4 \exp(i\omega t)$, about its major dimension, the X_1 axis and yaw, $\zeta_6 \exp(i\omega t)$; about its minor dimension, the X_3 axis, causing perturbations in the surrounding fluid.

We introduce a perturbed velocity potential Φ, such that the perturbed velocity (about the steady speed U) is $V_i=+\Phi_{,i}$ for $i=1,2,3$. We nondimensionalize the problem using the maximum length ℓ as a length scale, and ℓ/U as a time scale. This introduces reduced coordinates, $x_i=X_i/\ell$, reduced time, $\tau=tU/\ell$, as the new independent variables, a reduced perturbation velocity potential, $\phi=\Phi/U\ell$, as the new dependent

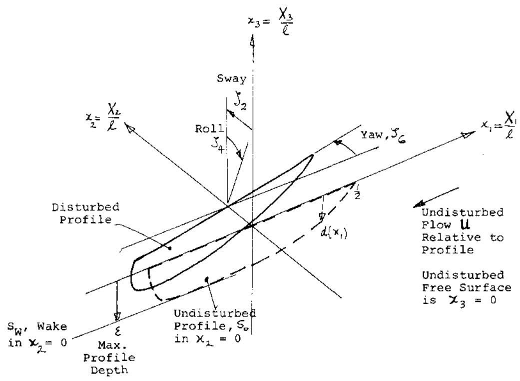

Fig. 1 Coordinate systems and definitions.

variable and a reduced frequency, $\nu=\omega\ell/U$, a draft to length ratio ϵ, and a Froude number, $F=U^2/g\ell$, as nondimensional parameters.

The governing equations are then

$$\phi_{,ii}=0 \text{ in } -\infty<x_3<0 \tag{1}$$

$$F(\ddot{\phi}-2\dot{\phi}_{,1}+\phi_{,11})+\phi_{,3}=0 \text{ on } x_3=0 \tag{2a}$$

the free surface kinematic condition

$$\phi_{,2}=\dot{\zeta}_2-x_3\dot{\zeta}_4+x_1\dot{\zeta}_6-\zeta_6 \tag{2b}$$

on the mean position of S, and

$$p=2(\partial/\partial x_1-i\nu)\phi \text{ (perturbation pressure)} \tag{2c}$$

With a suitable radiation condition, Eqs. (1) and (2) constitute a fully posed boundary value problem for the determination of ϕ, p, and generalized hydrodynamic coefficients for the motions ζ_j, ($j=2,4,6$). The problem is geometrically symmetrical about $x_2=0$, and for antisymmetric perturbation motions, ζ_2, ζ_4, ζ_6, the pressure and potential distributions must be antisymmetrical about $x_2=0$.

Define

$$\phi=\phi_2(\dot{\zeta}_2-\zeta_6)+\phi_4\dot{\zeta}_4+\phi_6\dot{\zeta}_6 \tag{3}$$

such that ϕ_β represents the potential for an oscillation in mode ζ_β with boundary conditions

$$\phi_{2,2}=1 \quad \phi_{4,2}=-x_3 \quad \phi_{6,2}=x_1 \tag{4}$$

each satisfying Laplace's equation in $x_3<0$ and appropriate free surface conditions.

The "doublet" distributions defined by

$$M_\beta=\phi_{\beta+}-\phi_{\beta-}\equiv R_\beta+i\nu I_\beta \tag{5}$$

must be such that[7] a) the potential is continuous at the leading edge ($x_1=x_\ell(x_3)$ or $x_3=-d(x_1)$) and therefore $M_\beta(x_1,-d(x_1))=0$, except at a straight leading edge $x_1=\frac{1}{2}$ when a discontinuity in M_β will exist; and b) across the wake the pressure is continuous, the linearized Bernoulli equation for harmonic motion of S_o then giving[4,7]

$$M_\beta(x_1,x_3)=M_\beta(x_t(x_3),x_3)\exp\{-i\nu(x_1-x_t(x_3))\} \tag{6}$$

(See for example Refs. 4 and 7).

Integral Equation Formulation

Solutions for M_β are approached through the use of Green's Functions in an integral equation formulation, and their approximation using expansions in inverse Froude number.

The Green's Function $G(x_i;y_i)e^{i\nu\tau}$ for a field point x_i, due to a unit source of oscillatory strength at a source point y_i satisfies

$$G_{,ii}(x_i;y_i;\nu) = -4\pi\delta(x_1-y_1)\delta(x_2-y_2)\delta(x_3-y_3)e^{i\nu\tau} \quad \text{in } (-\infty < {}^{x_3}_{y_3} < 0) \tag{7}$$

and a free surface boundary condition

$$\nu^2 G - 2\nu G_{,1} + G_{,11} + (1/F)G_{,3} = 0 \text{ on } x_3 = 0 \tag{8}$$

Green's theorem is applied for the potential ϕ_β and the Green's Function to a volume bounded by a) the ship surface S_o below the mean water line; b) the wake S_W behind the ship; (assumed in $x_2=0$), c) the free surface S_F of horizontal radius $\to\infty$; d) a small spherical surface surrounding a field point at x_i; and e) a closing hemispherical surface S_∞ in $x_3<0$ of radius $\to\infty$. The integral over the closing hemisphere vanishes by Riemann's Lemma, and the integral over the surface S_F may also be shown to vanish.[6]

By employing the usual maneuver of shrinking the field point sphere to a point, and noting antisymmetry aspects of the problem, an integral equation for M_β is obtained

$$\underset{x_2\to 0}{Lt} \int_{S_o+S_{W'}} M_\beta(y_1,y_o,y_3)\frac{\partial^2 G}{\partial y_2^2}(x_i;y_1,0,y_3)\,dy_1dy_3$$

$$= 4\pi n_{\beta+}(x_1,x_3) \tag{9}$$

in which

$$n_{2+} = 1 \quad n_{4+} = -x_3 \quad n_{6+} = x_1 \tag{10}$$

We shall concentrate on the effects of Froude number rather than frequency ν or draft-to-length ratio ϵ, and therefore consider ν to be small (quasisteady case) and ϵ to be small. Both of these parameters are important in actual applications and future study will include their influence. The governing equations however are set up such that all of these effects can be included.

Green's Function Expansion—High Speed

For high speed (large F) solutions, expansions of the integral term in powers of $K \equiv F^{-1/2}$ will be sought. Consider $\nu\to 0$, (the quasisteady case), for which $k_1 \to \sec^2\Theta/F = K^2\sec^2\Theta; k_2\to 0$. The limiting quasisteady form of the high speed Green's Function becomes

$$G(x_i;y_i) = \frac{1}{r_1} - \frac{1}{r_2} - \frac{4K^2}{\pi} \cdot Re\int_0^{\pi/2}\int_L \exp[k\{(x_3+y_3)+i(x_1-y_1)\cos\Theta\}]$$

$$\cdot \frac{\cos[k(x_2-y_2)\sin\Theta]}{(k-K^2\sec^2\Theta)}\sec^2\Theta\, dk\, d\Theta \tag{11}$$

where

$${}^{r_1^2}_{r_2^2} = (x_1-y_1)^2 + (x_2-y_2)^2 + (x_3 \mp y_3)^2$$

Then, differentiating twice with respect to y_2, for

$$x_2, y_2 = 0 \tag{12a}$$

$$\frac{\partial^2 G}{\partial y_2^2}(x_1,0,x_3;y_1,0,y_3) = -\frac{1}{r_1^3} + \frac{1}{r_2^3} + \frac{4K^2}{\pi}$$

$$\times Re\int_0^{\pi/2}\int_L \frac{k^2\sin^2\Theta\sec^2\Theta\exp[k\{(x_3+y_3)+i(x_i-y_i)\cos\Theta\}]}{(k-K^2\sec^2\Theta)}$$

$$\times\, dk\,d\Theta = {}^oG_{,22} + {}^RG_{,22}(K) \tag{12b}$$

where the limiting term

$${}^oG_{,22} = -\frac{1}{r_1^3} + \frac{1}{r_2^3} \tag{13}$$

and, the "remainder" term ${}^RG_{,22}$ may be written as

$${}^RG_{,22} = \frac{4K^2}{\pi} Re\int_0^{\pi/2}\sin^2\Theta\sec^2\Theta\frac{\partial^2 I(z)}{\partial x_3^2}d\Theta$$

$$= \frac{4K^2}{\pi} Re\int_0^{\pi/2}\sin^2\Theta\sec^2\Theta\frac{\partial^2}{\partial x_3^2}$$

$$\times\left[\left\{{}^{-2\pi i}_{\;\;0} + E_1(z)\right\}e^z\right]d\Theta, (y_1 \gtrless x_1) \tag{14}$$

where

$$z = K^2\sec^2\Theta\{(x_3+y_3)+i(x_i-y_1)\cos\Theta\} \tag{15}$$

With some rearrangement and defining a new variable

$$\eta = \sec^2\Theta - 1 \tag{16}$$

this may be written

$${}^RG_{,22} = \frac{2K^6}{\pi} Re\int_0^\infty(\eta^{3/2}+\eta^{1/2})e^z$$

$$\cdot\left[\left({}^{-2\pi i}_{\;\;0}\right) + \frac{2E_3(z)}{z^2}\right]d\eta, (y_1 \gtrless x_1) \tag{17}$$

Split this into "local" and "wave" terms, respectively, defined as

$${}^LG_{,22} = \frac{4K^6}{\pi} Re\int_0^\infty\frac{(\eta^{3/2}+\eta^{1/2}e^z)E_3(z)}{z^2}d\eta \text{ for } y_1 \gtrless x_1 \tag{18}$$

and

$${}^WG_{,22} = 4K^6\int_0^\infty(\eta^{3/2}+\eta^{1/2})\exp\{K^2(x_3+y_3)(1+\eta)\}$$

$$\sin\{K^2(x_1-y_1)(1+\eta)^{1/2}\}d\eta \quad \text{for } y_1 > x_1 \text{ only} \tag{19}$$

These will be considered separately. (It may be noted however that the wave term is identical to the low speed form.[6,7])

The Wave Term ${}^WG_{,22}(y_1>x_1$ *Only*)

By expanding the sine function as a power series, Laplace transforming into Kummer's functions, and expanding, ${}^WG_{,22}$ is shown to be dominated by [9]

$${}^WG_{,22} \sim \frac{8e^{-K^2\epsilon\gamma}K^2(x_1-y_1)}{(\epsilon\gamma)^3} \tag{20}$$

for $y_1>x_1$, $K^2<<1$, and $K^2\epsilon<1$.

The Local Term ${}^LG_{,22}(y \gtrless x_1)$)

Equation (21) may be developed into

$${}^LG_{,22} = \frac{4K^6}{\pi}\int_0^\infty(\eta^{3/2}+\eta^{1/2})e^{Re(z)}\left\{\frac{\psi(3)\cos(Im(z))}{2} + \frac{\Theta\sin(Im(z))}{2} - \frac{\ell n\chi_1\cos(Im(z))}{2} - \sum_{m=o}^\infty\frac{(-1)^m\chi^{m-2}\cos(Im(z)+(m-2)\Theta)}{(m-2)m!}\right\}d\eta \tag{21}$$

Expanding trigonometric functions in appropriate power series, and using various Laplace transforms ${}^{L}G_{,22}$ is shown for $K^2 << \epsilon\gamma$ to be dominated by[9]

$$ {}^{L}G_{,22} = \frac{{}^{1}CKe^{-K^2\epsilon\gamma}}{(\epsilon\gamma)^{5/2}} \quad (22)$$

where

$$ {}^{1}C = \{ -7/4 - 4C_2 + 3/2(\ell n(4\bar{\gamma}) - \bar{\gamma}) \} \pi^{-1/2} \quad (23)$$

and

$$ C_2/\pi^{1/2} = 1/\pi . \sum_{m=o}^{\infty} \frac{\Gamma(1/2+m)}{(m-2)\Gamma(m+1)}, (m \neq 2)$$

$$ = -\{ -1 + \sum_{m=3}^{\infty} \frac{(2m-1)!}{2^{2m-1}\cdot(m-2)m!(m-1)!} \} \cdot \pi^{-1/2} \quad (24)$$

Note that the ${}^{L}G_{,22}$ term is of order K^{-1} larger than the wave term ${}^{w}G_{,22}$.

Note on a Green's Function Expansion—Low Speed

Some reference to "low speed" results is included since certain features are interestingly related to the high speed case.[6,7] The wave and local terms, denoted again by ${}^{w}G_{,22}$ and ${}^{L}G_{,22}$, respectively, are treated in some detail in Ref. 9. The wave term ${}^{w}G_{,22}$ $(y_1 > x_1)$ is dominated by

$$ {}^{W}G_{,22} = \frac{2\cdot\pi^{1/2}K^3e^{K^2\epsilon\gamma}\cdot\sin[K^2(x_1-y_1)+(3/2)\cdot\tan^{-1}\{-(x_1-y_1)/2(x_3+y_3)\}]}{\{(x_3+y_3)^2+[(x_1-y_1)/2]^2\}^{3/4}} \quad (25)$$

The local term ${}^{L}G_{,22}$ can be expressed with a useful expansion for $e^zE_4(z)$. The result, that the local term is dominated by a term of $0(K^{-2})$ is obtained quite easily, and agrees with Ref. 6. In future work, this result should be useful for determining coefficients at Froude numbers of 0(1), when the local and wave terms may be of roughly the same importance.

General Solution Formulations (High Speed)

Use of Green's Function Expansions

The integral equation for the doublet distribution is of the form (Eq. 9).

$$ \int_{S_0+S_{w'}} M_\beta G_{,22} dS = N_\beta \, (=4\pi n_{\beta+}) \quad (26)$$

and

$$ G_{,22} = {}^{0}G_{,22} + {}^{R}G_{,22}$$

$$ \approx {}^{0}G_{,22} + (K^1\cdot{}^{1}G_{,22} + K^2\cdot{}^{2}G_{,22} + ..) \quad (27)$$

where the term $\exp(-K^2\epsilon\gamma)$ has been expanded in powers of K, assuming $K^2\epsilon << 1$.

If similar expansions for doublet distributions are assumed, then sequential boundary conditions and integral equations for ${}^{0}M$, ${}^{1}M$, ${}^{2}M$ are:

For ${}^{0}M_\beta$:

$$ \int_{S_0+S_{w'}} {}^{0}M_\beta {}^{0}G_{,22} dS = N_\beta = 4\pi n_{\beta+} \text{ in } x_3 < 0 \quad (28a)$$

with

and therefore $\left.\begin{matrix} {}^{0}G_{,11} = 0 \\ {}^{0}M_{,11} = 0 \end{matrix}\right\} \text{on } x_3 = 0$ (28b) (28c)

For ${}^{1}M_\beta$:

$$ \int_{S_0+S_{w'}} {}^{1}M_\beta {}^{0}G_{,22} dS$$

$$ = -\int_{S_0+S_{w'}} {}^{0}M_\beta {}^{1}G_{,22} dS \equiv -{}^{1}N_\beta, \text{in } x_3 < 0 \quad (29a)$$

with

$$ {}^{1}G_{,11} = 0 \text{ on } x_3 = 0 \quad (29b)$$

and therefore $\quad {}^{1}M_{,11} = 0$ (29c)

For ${}^{2}M_\beta$:

$$ \int_{S_0+S_{w'}} {}^{2}M\cdot{}^{0}G_{,22} dS = -\int_{S_0+S_{w'}} ({}^{0}M_\beta\cdot{}^{2}G_{,22} + {}^{1}M_\beta \cdot {}^{1}G_{,22}) dS \text{ in } x_3 < 0 \quad (30a)$$

with $\left.\begin{matrix} {}^{2}G_{,11} + {}^{0}G_{,3} = 0 \\ {}^{2}M_{,11} + {}^{0}M_{,3} = 0 \end{matrix}\right\} \text{on } x_3 = 0$ (30b)
and therefore (30c)

Approximation for Integral Kernel ($\epsilon < 1$)

The left-hand side of the integral equation for any order of M is typically

$$ \int_{-\infty}^{1/2}\int_{-d(y_1)}^{0} M(y_1,y_3)\,{}^{0}G_{,22}(x_1,0,x_3;y_1,0,y_3)\,dy_3 dy_1$$

$$ = \int_{-\infty}^{1/2}\int_{-d(y_1)}^{0} M(y_1,y_3)\{-[(x_1-y_1)^2+(x_3-y_3)^2]^{-3/2} + [(x_1-y_1)^2+(x_3+y_3)^2]^{-3/2}\} dy_3 dy_1 \quad (31)$$

Assuming $(x_3 \pm y_3)^2 \ll (x_1-y_1)^2$ in the wake, integrating by parts with respect to y_3, noting $M(y_1, -d) = 0$, and manipulating algebraically results in

$$ \int M^0 G_{,22} dS$$

$$ \doteq 2x_3 \int_{-1/2}^{1/2} M(y_1,o)(x_1-y_1)^{-2}\{(x_1-y_1)^2 + x_3^2\}^{-1/2}\cdot dy_1$$

$$ +2\int_{-d(x_1)}^{0} \frac{\partial M(x_1,y_3)}{\partial y_3}\{(x_3-y_3)^{-1} + (x_3+y_3)^{-1}\} dy_3$$

for $\epsilon \ll 1$ (32)

assuming that $(\partial M/\partial y_3)$ is continuous in x_1.

If use is made of the properties that ${}^{0}G$ is antisymmetric in y_3, is zero in $x_3 = 0$ for all y_3, satisfies Laplace's equation, and the problem is extended in $x_3 > 0$ such that ${}^{0}G$ is ODD in x_3 and y_3, then Eq. (32) for some order of M, ${}^{\alpha}M$ say, in $x_3 < 0$, can be extended in x_3 to give

$$ 2\,⨍_{-d(x_1)}^{d(x_1)} \frac{\partial^{\alpha}M}{\partial y_3}(x_1,y_3)\cdot\frac{dy_3}{(x_3-y_3)}$$

$$ = {}^{\alpha}N(x_1,x_3) - 2x_3\int_{-1/2}^{1/2} \frac{{}^{\alpha}M(y_1,0)dy_1}{(x_1-y_1)^2\{(x_1-y_1)^2+x_3^2\}^{1/2}} \quad (33)$$

in which $(\partial^{\alpha}M/\partial y_3)$ is EVEN in x_3 and ${}^{\alpha}N$, ${}^{\alpha}M$ are ODD in x_3, (not necessarily continuous at $x_3 = 0$). Equation (33) is an integral equation for $(\partial^\alpha M/\partial y_3)$ the appearance of which suggests iterative solutions. For zero and first-order solutions, further simplifications are possible which enable direct use of the Sohngen integral inversion theorem[8] which states that

If $$\int_{-d}^{d} \frac{f(y)}{(x-y)}\, dy = g(x) \tag{34a}$$

and $$\int_{-d}^{d} f(y)dy = 0 \tag{34b}$$

then $$f(y) = \frac{-1}{\pi^2 (d^2-y^2)^{1/2}} \int_{-d}^{d} \frac{g(x)\,(d^2-x^2)^{1/2}}{(y-x)}\, dx \tag{34c}$$

Simplification of Higher-Order Equations

Reconsider the general integral equation formulation, Eq. (26), for arbitrary modal motion,

$$\int_{S_0+S_{w'}} M(y_1,0,y_3)\,G_{,22}(x_1,0,x_3;y_1,o,y_3)\,dy_1 dy_3 = N(x_1,x_3) \tag{35}$$

Expressing again

$$M(x_1,0,x_3) = {}^0M + {}^RM(K) \tag{36a}$$

$$G_{,22}(x_1 0,x_3;y_1,o,y_3) = {}^0G + {}^RG(K) \tag{36b}$$

Then Eq. (35) is satisfied for all K if

$$\int_{S_0+S_{w'}} {}^0M^0G_{,22}dy,dy_3 = N(x_1,x_3) \tag{37}$$

and

$$\int_{S_0+S_{w'}} \{ {}^0M^RG_{,22} + {}^RM^0G_{,22} + {}^RM^RG_{,22} \} dy_1 dy_3 = 0 \tag{38}$$

Ignoring higher-order contributions from ${}^RM^RG$, using Eq. (32) and formally inverting, Eq. (38) may be developed to

$$\frac{\partial^R M(x_1,\dot{y}_3)}{\partial y_3} = \frac{-1}{\pi^2 (d^2(x_1)-x_3^2)^{1/2}} \int_{-d(x_1)}^{d(x_1)} \frac{(d^2(x_1)-x_3^2)^{1/2}}{(y_3-x_3)} \cdot \Big[-x_3 \int_{-1/2}^{1/2} \frac{{}^RM(y_1,o)dy_1}{(x_1-y_1)^2 \{(x_1-y_1)^2+x_3^2\}^{1/2}} - 1/2 \int_{S_0+S_{w'}} {}^0M(y_1,\xi_3).{}^RG_{,22}(x_1,x_3;y_1,\xi_3)dy_1 d\xi_3 \Big] dx_3 \tag{39}$$

In the right-hand side inner integrals, ${}^0M$ will have been determined on S_0 from previous solutions, and will be constant in $S_{w'}$. If ${}^RG$ is split into ${}^LG$ and ${}^WG$ (the latter wave term being zero is $S_{W'}$) the $S_{w'}$ contribution to the right side of Eq. (39) may be developed in a manner very similar to the ${}^LG_{,22}$ expansion.

The result from

$$\int_{-\infty}^{1/2} {}^LG_{,22}dy_1 \equiv {}^LJ_{,22} \tag{40}$$

is dominated by

$${}^LJ_{,22} \approx \frac{{}^1C_w K e^{-K^2\epsilon\bar{\gamma}}.(x_1+1/2)}{(\epsilon\bar{\gamma})^{5/2}} \tag{41}$$

in which

$$\epsilon\bar{\gamma} = -\epsilon(\bar{x}_3+\bar{\xi}_3) \tag{42}$$

$${}^1C_w = \{ -3/2 - 4^wC_2 + 3/2\cdot(\ln 4\bar{\gamma} - \bar{\gamma}) \}.\pi^{-1/2} \tag{43}$$

On substituting this back into Eq. (42) a simpler equation for ${}^RM$ is obtained:

$$\frac{\partial^R M(x_1,y_3)}{\partial y_3} = -\frac{1}{\pi^2(d^2(x_1)-y_3^2)^{1/2}} \int_{-d(x_1)}^{d(x_1)} \frac{(d^2(x_1)-x_3^2)^{1/2}}{(y_3-x_3)} \cdot \Big[-x_3 \int_{-1/2}^{1/2} \frac{{}^RM(y_1,0)dy_1}{(x_1-y_1)^{1/2}\{(x_1-y_1)^2+x_3^2\}} - 1/2\cdot \int_{-1/2}^{1/2}\int_{-d(y_1)}^{0} {}^0M(y_1,\xi_3)\,{}^RG_{,22}(x_1,x_3;y_1,\xi_3)dy_1 d\xi_3 - 1/2\cdot \int_{d_t}^{0} {}^0M(-1/2,\xi_3)\,{}^LJ_{,22}(x_1,x_3;\xi_3)d\xi_3 \Big] dx_3 \tag{44}$$

Solutions for Doublet Distributions and Hydrodynamic Coefficients

Zero-Order Solution ${}^0M_\beta$

Consider the integral equation (33) (for $\alpha=0$). Since ${}^0G$ is zero on $x_3,y_3=0$, ${}^0M$ will also be, and the equation reduces to

$$\int_{-d(x_1)}^{d(x_1)} \frac{\partial^0 M_\beta(x_1,y_3)}{\partial y_3} \cdot \frac{dy_3}{(x_3-y_3)} = 2\pi n_{\beta+}(x_1,x_3) \tag{45}$$

in which

$$n_{\beta+} = \left.\begin{matrix} \pm 1 \\ -x_3 \\ \pm x_1 \end{matrix}\right\} \text{ in } \left\{\begin{matrix} -d<x_3<0 \\ 0<x_3<d \\ \end{matrix}\right\},\ \beta = \left\{\begin{matrix} 2 \\ 4 \\ 6 \end{matrix}\right. \tag{46}$$

Inverting, using Eq. (34)

$$\frac{\partial^0 M_\beta}{\partial y_3} = \frac{-2}{\pi(d^2(x_1)-y_3^2)^{1/2}} \int_{-d(x_1)}^{d(x_1)} n_{\beta+}(x_1,x_3) \cdot (d^2(x_1)-x_3^2)^{1/2} \cdot (y_3-x_3)^{-1} dx_3 \tag{47}$$

Assuming a convex profile, Eq. (47) gives

$$\frac{\partial^0 M_2(y_1,y_3)}{\partial y_3} = \frac{1}{y_1}\cdot\frac{\partial^0 M_6}{\partial y_3} = \frac{4}{\pi} \cdot \left\{ \frac{d(y_1)}{(d^2(y_1)-y_3^2)^{1/2}} - \ln\left| \frac{(d^2(y_1)-y_3^2)^{1/2}+d(y_1)}{-y_3} \right| \right\}$$

and

$${}^0M_2(y_1,y_3) = \frac{1}{y_1}\cdot {}^0M_6(y_1,y_3) = \frac{-4y_3}{\pi} \ln\left| \frac{(d^2(y_1)-y_3^2)^{1/2}+d(y_1)}{-y_3} \right| \tag{48}$$

and

$$\frac{\partial^0 M_4(y_1,y_3)}{\partial y_3} = -\frac{(d^2(y_1)-2y_3^2)}{(d^2(y_1)-y_3^2)} \tag{49a}$$

$${}^0M_4(y_1,y_3) = \frac{1}{\pi^2(d^2(x_1)-y_3^2)^{1/2}} \tag{49b}$$

These are sketched in Fig. 2, with the corresponding low speed solutions.[6] The doublet distributions represent effects of modal motion of the profile with an "image" profile performing anti-symmetric (symmetric[6]) motions with respect to

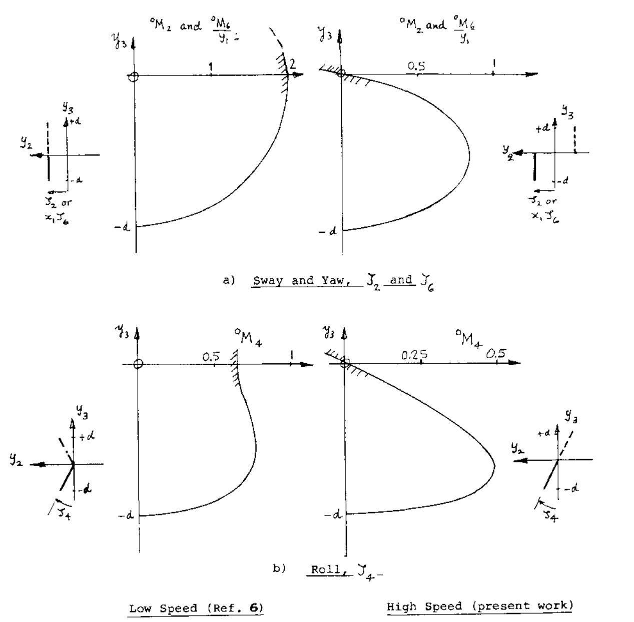

Fig. 2 Zero-order doublet distributions $0M\beta$ and 'extended' boundary conditions in $-d<y3<d$.

the $x_3,y_3=0$ plane. The sketches with the $^0M_\beta$ curves represent the quasisteady boundary conditions implied in the extended x_3 domain.

Zeroth-Order Generalized Hydrodynamic Coefficients

Assume a trailing edge of depth d_t, normal to $x_3=0$, and define dimensionless generalized hydrodynamic coefficients by

$$Q_i=\frac{\partial W/\partial\zeta_i}{\frac{1}{2}\rho U^2\ell^3}=A_{ij}\ddot{\zeta}+D_{ij}\dot{\zeta}+K_{ij}\zeta=(-\nu^2A_{ij}+i\nu D_{ij}+K_{ij})\zeta \tag{50}$$

For a convex profile of arbitrary depth distribution the doublet distributions of Eq. (49) integrated over y_1,y_3 (and to y_ℓ^+ to include the leading edge doublet discontinuity) give hydrodynamic coefficients as follows, for $i,j=2,4,6$:

$$[K_{ij}]=\begin{bmatrix} 0 & 0 & -(4/\pi)d_t^2 \\ 0 & 0 & -(2/\pi)d_t^3 \\ 0 & 0 & -(4/\pi)\int_{-1/2}^{1/2} d^2(x_1)dx_1+(2/\pi)d_t^2 \end{bmatrix} \tag{51}$$

$$[D_{ij}]=\begin{bmatrix} (4/\pi)d_t^2 & (2/\pi)d_t^3 & -(4/\pi)\int_{-1/2}^{1/2} d^2(x_1)dx_1-(2/\pi)d_t^2 \\ (2/\pi)d_t^3 & (\pi/8)d_t^4 & -(2/3)\int_{-1/2}^{1/2} d^3(x_1)dx_1-(d_t^3/\pi) \\ (4/\pi)\int_{-1/2}^{1/2} d^2(x_1)dx_1-(2/\pi)d_t^2 & (2/3)\int_{-1/2}^{1/2} d^3(x_1)dx_1-(d_t^3/3) & d_t^3/\pi \end{bmatrix} \tag{52}$$

$$[A_{ij}]=\begin{bmatrix} (4/\pi)\int_{-1/2}^{1/2} d^2(x_1)dx_1 & (2/3)\int_{-1/2}^{1/2} d^3(x_1)dx_1 & (4/\pi)\int_{-1/2}^{1/2} x_1d^2(x_1)dx_1 \\ (2/3)\int_{-1/2}^{1/2} d^3(x_1)dx_1 & (\pi/8)\int_{-1/2}^{1/2} d^4(x_1)dx_1 & (2/3)\int_{-1/2}^{1/2} x_1d^3(x_1)dx_1 \\ (4/\pi)\int_{-1/2}^{1/2} x_1d^2(x_1)dx_1 & (2/3)\int_{-1/2}^{1/2} x_1d^3(x_1)dx_1 & (4/\pi)\int_{-1/2}^{1/2} x_1^2d^2(x_1)dx_1 \end{bmatrix} \tag{53}$$

For a rectangular profile of consta.nt depth, d

$$[K_{ij}]=\begin{bmatrix} 0 & 0 & -(4/\pi)d^2 \\ 0 & 0 & -(2/\pi)d^3 \\ 0 & 0 & -(2/\pi)d^2 \end{bmatrix} \tag{54}$$

$$[D_{ij}]=\begin{bmatrix} (4/\pi)d^2 & (2/\pi)d^3 & -(6/\pi)d^2 \\ (2/\pi)d^3 & (\pi/8)d^4 & -d^3 \\ (2/\pi)d^2 & d^3/3 & d^2/\pi \end{bmatrix} \tag{55}$$

$$[A_{ij}]=\begin{bmatrix} (4/\pi)d^2 & (2/3)d^3 & 0 \\ (2/3)d^3 & (\pi/8)d^4 & 0 \\ 0 & 0 & d^2/3\pi \end{bmatrix} \tag{56}$$

Equations for First-Order Distribution, $^1M_\beta$

From Eqs. (22), (39) and (41)

$$\frac{\partial\,^1M(x_1,y_3)}{\partial y_3}=\frac{-1}{\pi^2(d^2-y_3^2)^{1/2}}⨍_{-d(x_1)}^{d(x_1)}\frac{(d^2(x_1)-x_3^2)^{1/2}}{(y_3-x_3)}\cdot\Big[-x_3\int_{-1/2}^{1/2}\frac{^1M(y_1,0)dy_1}{(x_1-y_1)^2\{(x_1-y_1)^2+x_3^2\}^{1/2}}$$
$$-\tfrac{1}{2}\int_{S_0}{}^0M(y_1,\xi_3)\cdot{}^1G_{,22}(x_1,x_3;y_1,\xi_3)dy_1d\xi_3$$
$$-\tfrac{1}{2}\int_{-d_t}^{0}{}^0M(-\tfrac{1}{2},\xi_3)\cdot{}^1J_{,22}(x_1,x_3;\xi_3)\Big]dx_3 \tag{57}$$

The boundary condition on ^1G and hence ^1M is

$$G,^1_{11}=0 \quad \text{on} \quad x_3=0 \tag{58}$$

Thus, on the free surface ^1G is at most linear in x_1 and for proper behavior at large x_1,y_1, must therefore be zero on the free surface. A similar argument applies to ^1M. Again, then the inner line integral over y_1, for $x_3=0$ vanishes.

Table 1 Comparison of local and wave effects

$0(G,22)$	Low Froude number $(F<<1, K>>1)$	High Froude number $(F>>1, K<<1)$
Wave term	$k^3\exp(-K^2x)$	$K^2\exp(-K^2x)$
Local term	$K^{-2}\exp(-K^2x)$	$K\exp(-K^2x)$

Using the expressions for ${}^0M_\beta$ from Eq. (49)

$$\frac{\partial\, {}^1M_\beta(x_1,y_3)}{\partial y_3} = \frac{K}{2\pi^2(d^2-y_3^2)^{1/2}} \int_{-d(x_1)}^{d(x_1)} \frac{(d^2(x_1)-x_3^2)^{1/2}}{(y_3-x_3)} \cdot {}^1C \begin{Bmatrix} -4/\pi \\ -1 \\ -4/\pi \end{Bmatrix} \int_{-1/2}^{1/2} \begin{Bmatrix} 1 \\ 1 \\ y_1 \end{Bmatrix} dy_1 \int_{-d(y_1)}^{0} \begin{Bmatrix} \xi_3 \operatorname{sech}^{-1}(-\xi_3/d(y_1)) \\ \xi_3(d_3^2-\xi_3^2)^{1/2} \\ \xi_3 \operatorname{sech}^{-1}(-\xi_3/d(y_1)) \end{Bmatrix}$$

$$+ {}^1C_W(x_1+1/2) \begin{Bmatrix} -4/\pi \\ -1 \\ 2/\pi \end{Bmatrix} \int_{-d_t}^{0} \begin{Bmatrix} \xi_3 \operatorname{sech}^{-1}(-\xi_3/d_t) \\ \xi_3(d_t^2-\xi_3^2)^{1/2} \\ \xi_3 \operatorname{sech}^{-1}(-\xi_3/d_t) \end{Bmatrix} \{-(x_3+\xi_3)\}^{-5/2} \cdot d\xi_3 dx_3 \tag{59}$$

In Eq. (59), for small enough K, exp $(-K^2\epsilon\gamma)$ has been assumed ~ 1. The columns represent modes $\beta=2,4,6$ (sway, roll, yaw) sequentially from top to bottom.

The function of (x_3,y_1) resulting from the ξ_3 integration, defined in $-d(x_1)<x_3<0$, is to be extended antisymmetrically into the range $-d(x_1)<x_3<d(x_1)$. After integrations with respect to ξ_3, y_1, and x_3, the application of boundary conditions on ${}^1M_\beta$ and further integrations lead to the first-order hydrodynamic coefficients. The development of Eq. (59) for arbitrary profiles is the subject of a subsequent paper.

Conclusions

The hydrodynamic coefficients for oscillatory lateral motions of thin profiles travelling at high steady Froude number have been examined. These are necessary for vessel and structural stability, and control investigations on projected high speed surface effect ships. Useful formulations of Green's Functions in integral form and in series expansion form have been determined for both high and low Froude numbers.9.

For high Froude numbers, the "local" effect of the motions is of more importance than the "wave" effect in contradistinction to the low Froude number case. This is summarized in Table 1.

Explicit expressions are given for the zero-order high speed hydrodynamic coefficients for arbitrary profiles. At typical highest speeds (at which hydroelastic effects will be most important), the zero-order terms may give sufficient engineering accuracy. Implicit expressions are also given for the next order doublet distributions. These are under development, and will help determine the range of applicability of the zeroth-order solution. They will be the subject of a subsequent paper.

The results obtained require the draft to length ratio to be such that $K2=F-1<<\epsilon<<1$, and the reduced frequency to be "zero" (quasisteady case). SES applications reasonably require $K2\sim 0(\epsilon)<<1$, and a dual expansion in terms of both K and ϵ would appear appropriate. Beyond this modification, the consideration of finite reduced frequencies and thickness appears to be the next step in improving the applicability of the results. Inclusion of distortional modes is also necessary but poses no formal difficulty. Finally, experimental verification of the theory is also desirable to illustrate and account for omission of important cavitation and viscous effects. Clearly the work reported here represents only one piece in a long chain of problems aimed at determining and incorporating the significance of these effects in SES stability, loads, and structural dynamic criteria.

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